Appendix II. Rock-phsycics relationships

The velocity and density of a porous medium are influenced by the fluids that are present in the pore space. The bulk density ρ as a function of porosity ϕ is formulated in the following equation:

$$\rho = (1-\phi)\rho_s + \phi\rho_f , \qquad \text{(II.1)}$$

where:
ρ_s denotes the density of the solid fraction and ρ_f the density of the pore fluid.

The relationship between velocity, porosity and fluid content is more complicated. Willie's time average equation, or (empirical) extensions to this formula have been used by many workers (e.g. de Haas, 1992). Willie's equation (Wyllie et.al., 1958) is formulated as:

$$t = (1-\phi)t_s + \phi t_f , \qquad \text{(II.2)}$$

where t denotes sonic travel time of the rock, t_s the travel time in the solid matrix (i.e. empty porous rock), t_f travel time for the pore fluid and ϕ is the porosity.

This equation and the empirical extensions thereof, are not very reliable when used as fluid replacement algorithms, especially not for the gas-fill replacements.

The most widely used rock-physics models for studying wave propagation effects in porous media are the theoretical Gassman and Biot-Gassmann equations (see, e.g. Crans and Berkhout). The main difference between the two models is that Gassmann is applicable to low seismic frequencies only while Biot-Gassmann is also used for predicting frequency dependent velocities. The low frequency limit of Biot-Gassmann equals the Gassmann's equation which is formulated as:

$$c_p = \sqrt{\frac{\kappa_s}{\rho}\left(3\left(\frac{1-\sigma_b}{1+\sigma_b}\right)\beta + \frac{(1-\beta)^2}{1-\beta+\phi\left(\kappa_s/\kappa_f - 1\right)}\right)} , \qquad \text{(II.3)}$$

where:
c_p denotes the seismic velocity for compressional waves. For an explanation of the other symbols see Table II.1.

Table II.1 Rock and pore parameter definitions.

Parameter	Description	Unit
κ_s	compressibility modulus solid	N/m^2
κ_f	compressibility modulus fluid	N/m^2
σ_b	Poisson ratio	-
ϕ	Porosity	-
ρ_s	density solid	kg/m^3
ρ_f	density fluid	kg/m^3
ρ	density bulk	kg/m^3
c_p	P-wave velocity	m/s

Direct application of this equation to calculate rock velocities is of limited use since the Poisson ratio σ_b and the compressibility moduli κ_f , κ_s are, in general unknown. If, however, the velocity of a rock with a given saturation is known, then the Gassmann equation can be used to calculate the velocity of the same rock with a different saturation, as follows.

Assume that κ_w, κ_{hc}, s_w, s_{hc}, σ_b, ρ_s, ρ_w and ρ_{hc} are input parameters specified by the user. For a description see Table II.1; the index *hc* denotes hydrocarbons. First calculate the porosity ϕ using (II.1) for the brine-filled case. Then calculate ρ_f using s_w, s_{hc}, ρ_w and ρ_{hc}. Now calculate the density of the hydrocarbon-filled case using (II.1). Use Wood's law for the compressibility modulus of the fluid mixture κ_f. Wood's law is formulated as:

$$1/\kappa_f = s_w/\kappa_w + s_{hc}/\kappa_{hc} \, . \tag{II.4}$$

Now the Gassmann equation (II.3) can be employed to calculate the velocity of the hydrocarbon-filled rock. The Gassmann equation, as a fluid replacement algorithm is applied in two steps. In the first step, the frame strength, or Biot coefficient β defined as κ_m/κ_s where κ_m is the compressibility modulus of the matrix, is derived from the sound velocity of the brine-filled rock. Defining γ as:

$$\gamma = 3(1-\sigma_b)/(1+\sigma_b) \, , \tag{II.5}$$

and B as:

$$B = \phi(\kappa_s/\kappa_f - 1) \, , \tag{II.6}$$

then β can be calculated as:

$$\beta = 1 - A \pm \sqrt{(A+B)^2 - (B^2/(1-\gamma))}\,,$$
(II.7)

with:

$$A = ((\rho c_p^2/\kappa_s) + \gamma(B-1))/2(1-\gamma).$$
(II.8)

In the second step of the fluid replacement algorithm the assumption is made that ϕ, β and σ_b are independent from the fluid properties. Substitution of these variables together with the properties of the fluid mixture in (II.3), yields the velocity of the hydrocarbon filled rock. The sonic travel time follows as the reciprocal of this velocity.

As stated above, the Gassmann equation assumes the velocity to be independent from frequency. Biot (1956b) has proved, however, that velocity does depend on frequency. At low (seismic) frequencies this effect can in general be ignored. Anderson (1984) proved that this effect can be significant in special cases, e.g. low permeability rocks with low saturation gas in the pores.

Krief (1990) suggested that the Biot-Gassmann equation is simplified if the ratio of the shear modulus and the compressional modulus are constant for a given rock type. Based on this premise Krief derived the following equation that relates the compressional velocity to rock properties (SPT, 1992):

$$C_p^2 = (1-\beta)(k_s + \frac{4}{3}\mu_s) + \beta^2 \left| \frac{1}{\frac{(\beta-\phi)}{k_s} + \frac{\phi}{k_f}} \right|^{\frac{1}{\rho}}$$
(II.9)

where:
μ_s is the shear wave modulus of the solid. For the other parameters see Table II.1.

The shear velocity V_s is given by Krief as:

$$V_s^2 = \mu_s(1-\beta)\frac{1}{\rho}$$
(II.10)

In these equations all variables except the frame strength (or Biot coefficient) β are derived from log analysis. The Biot coefficient can be calculated when the compressional velocity is known. Alternatively, an empirical Biot coefficient related to (total) porosity can be used to calculate shear and compressional velocity. This empirical Krief equation is given by:

$$\beta = 1 - (1-\phi)^{\left(3/_{1-\phi}\right)}$$
(II.11)

Fluid replacement modelling plays an important role in the study of AVO anomalies. This is a seismic reservoir characterisation technique in which amplitude variations as a function of offset are studied on pre-stack seismic data. AVO studies are of particular interest in the exploration for gas. Several successful AVO case studies, aimed at predicting gas-fill, have been reported in literature (e.g. Allen et. al., 1993). The theory behind AVO exploration for gas is based on the differences in the response of both compressional (P-waves) and shear-waves (S-waves) of a porous reservoir rock, depending on its gas-saturations. Even a relatively low gas-saturation will substantially lower the P-wave velocities, whilst the S-wave velocities will be relatively unaffected. The ratio of P-wave velocity to S-wave velocity is an important factor in the partitioning of an incident P-wave when it strikes an interface. Thus, a change in amplitude can be expected along a reflector (i.e. as a function of offset) depending on the gas-fill. For some reservoirs the reflections associated with gas-bearing rock increase in amplitude with offset relative to other reflections. Such an increase with offset is anomalous; most reflections decrease in amplitude with offset. Most AVO studies try to detect such anomalies.

AVO effects can be modelled with the Zoeppritz equations. A popular approximation of these equation has been derived by Shuey (1985). The two-term equation represents the angular dependence of P-wave reflection coefficients with two parameters: the AVO intercept time *A* and the AVO gradient ***B*.** In practice (Gastagna and Swan, 1997), the AVO intercept is a band-limited measure of the normal incidence amplitude, while the AVO gradient is a measure of amplitude variation with offset. Assuming appropriate amplitude calibration, A is the normal incidence reflection coefficient and *B* is a measure of offset-dependent reflectivity. Shuey's equation is written as:

$$R(\theta) = A + B\sin^2(\theta)$$
(II.12)

where:
R indicates the reflection coefficient and θ the angle of incidence.